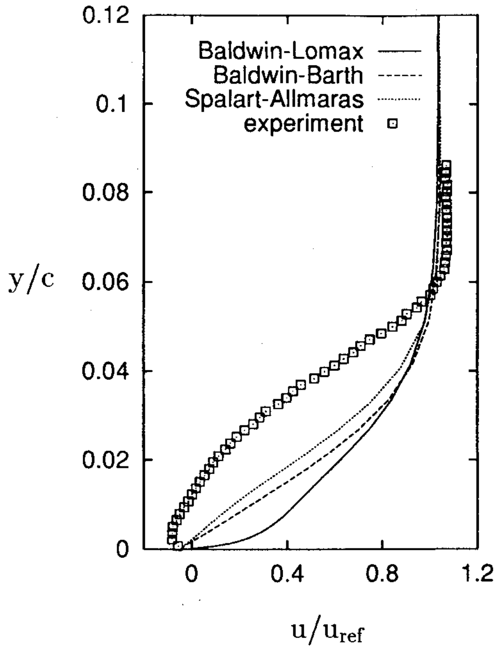

Fig. 4 Mean velocity profiles, $\alpha = 9.2$ deg, $x/c = 0.950$.

tics of the three films are evident. The improvements in the velocity magnitude errors resulting from the asymmetry correction for a typical calibration are shown in Table 1.

Results

The triple-split hot-film probe was used to measure mean velocities near the trailing edge of a 17% thick airfoil with moderate aft loading at two angles of attack, $\alpha = 0$ and $\alpha = 9.2$ deg. At $\alpha = 0$ deg, the flow is fully attached, whereas at $\alpha = 9.2$ deg there is a region of separated flow extending over approximately 20% chord. The tests were run at a chord-based Reynolds number of roughly 1.7×10^6 and a Mach number of 0.15. Transition was fixed at 8% chord on both surfaces, although it undoubtedly occurs earlier on the upper surface at the higher angle of attack.

Figure 3 shows the measured mean velocity profile in the boundary layer on the upper surface of the airfoil at $\alpha = 0$ deg. Also shown are computational results obtained using the well-established Navier-Stokes code ARC2D[9] with three different turbulence models, the Baldwin-Lomax model, the Baldwin-Barth model, and the Spalart-Allmaras model. Agreement between the measured and computed values is quite typical of attached adverse pressure gradient flows. Similar results for $\alpha = 9.2$ deg are shown in Fig. 4. The computed results drastically underestimate the thickness of the separated region. This is also typical for this flow solver, as discussed, for example, by Nelson et al.[10]

Conclusions

The calibration procedure for triple-split hot-film probes has been extended for probe asymmetry effects, leading to a substantial reduction in velocity magnitude errors. The probe has been used to measure mean velocity components in attached and separated two-dimensional turbulent flows about an airfoil. The results are sufficiently promising to justify further investigation of the probe.

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Nonlinear Thermal Dynamic Analysis of Graphite/Aluminum Composite Plates

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Introduction

BECAUSE of the increased application of composite materials in high-temperature environments, the thermoelastic analysis of laminated composite structures is important. Many researchers have applied the classical lamination theory to analyze laminated plates under thermomechanical loading,[1] which neglects shear deformation effects. The transverse shear deformation effects are not negligible as the ratios of inplane elastic modulus to transverse shear modulus are relatively large for fiber-reinforced composite laminates. The application of first-order shear deformation theory for the thermoelastic analysis of laminated plates has been reported by only a few investigators.[2–4] Reddy and Hsu[2] have considered the thermal bending of laminated plates. The analytical and finite element solutions for the thermal buckling of laminated plates have been reported by Tauchert[3] and Chandrashekhara,[4] respectively. However, the first-order shear deformation theory, based on the assumption of constant distribution of transverse shear through the thickness, requires a shear correction factor to account for the parabolic shear strain distribution. Higher order theories have been proposed which eliminate the need for a shear correction factor.[5] In the present work, nonlinear dynamic analysis of laminated plates subjected to rapid heating is investigated using a higher order shear deformation theory. A C^0 finite element model with seven degrees of freedom per node is implemented and numerical results are presented for laminated graphite/aluminum plates.

Mathematical Formulation

A laminated composite plate having length a, width b, and thickness h is considered. The higher order displacement field assumed can be expressed as

$$\begin{aligned} u(x,y,z,t) &= u_0(x,y,t) + z\psi_x(x,y,t) + z^3\phi_x(x,y,t) \\ v(x,y,z,t) &= v_0(x,y,t) + z\psi_y(x,y,t) + z^3\phi_y(x,y,t) \\ w(x,y,z,t) &= w_0(x,y,t) \end{aligned} \tag{1}$$

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where u, v, and w are displacements along the x, y, and z axes; u_0, v_0, and w_0 are the midplane displacements; and ψ_x, ψ_y, ϕ_x, and ϕ_y are the rotations.

The nonlinear strain-displacement relations, based on von-Kármán assumptions, can be written as

$$\varepsilon_x = \varepsilon_x^0 + z\kappa_x + z^3\eta_x, \quad \varepsilon_y = \varepsilon_y^0 + z\kappa_y + z^3\eta_y$$
$$\gamma_{yz} = \gamma_{yz}^0 + z^2\zeta_{yz}, \; \gamma_{xz} = \gamma_{xz}^0 + z^2\zeta_{xz}, \; \gamma_{xy} = \gamma_{xy}^0 + z\kappa_{xy} + z^3\eta_{xy} \tag{2}$$

where

$$\varepsilon_x^0 = \frac{\partial u_0}{\partial x} + \frac{1}{2}\left(\frac{\partial w_0}{\partial x}\right)^2, \quad \kappa_x = \frac{\partial \psi_x}{\partial x}, \quad \eta_x = \frac{\partial \phi_x}{\partial x}$$
$$\gamma_{yz}^0 = \psi_y + \frac{\partial w_0}{\partial y}, \quad \zeta_{yz} = 3\phi_y$$
$$\varepsilon_y^0 = \frac{\partial v_0}{\partial y} + \frac{1}{2}\left(\frac{\partial w_0}{\partial y}\right)^2, \quad \kappa_y = \frac{\partial \psi_y}{\partial y}, \quad \eta_y = \frac{\partial \phi_y}{\partial y}, \quad \gamma_{xz}^0 = \psi_x \tag{3}$$
$$\gamma_{xz}^0 = \psi_x + \frac{\partial w_0}{\partial x}, \quad \zeta_{xz} = 3\phi_x$$
$$\gamma_{xy}^0 = \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} + \frac{\partial w_0}{\partial x}\frac{\partial w_0}{\partial y}, \quad \kappa_{xy} = \frac{\partial \psi_x}{\partial y} + \frac{\partial \psi_y}{\partial x}$$
$$\eta_{xy} = \frac{\partial \phi_x}{\partial y} + \frac{\partial \phi_y}{\partial x}$$

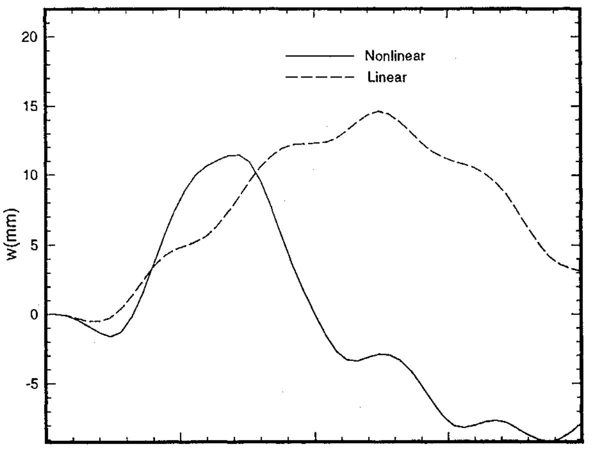

Fig. 1 Linear and nonlinear transient responses of a [0/90 deg] cross-ply plate.

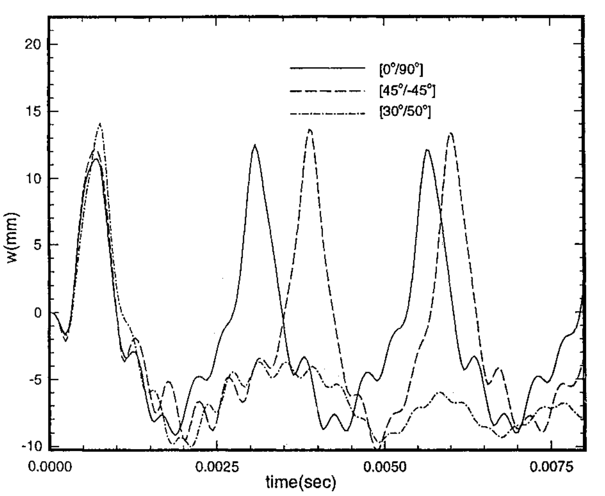

Fig. 2 Effect of the ply orientation on the nonlinear transient response of simply supported plates.

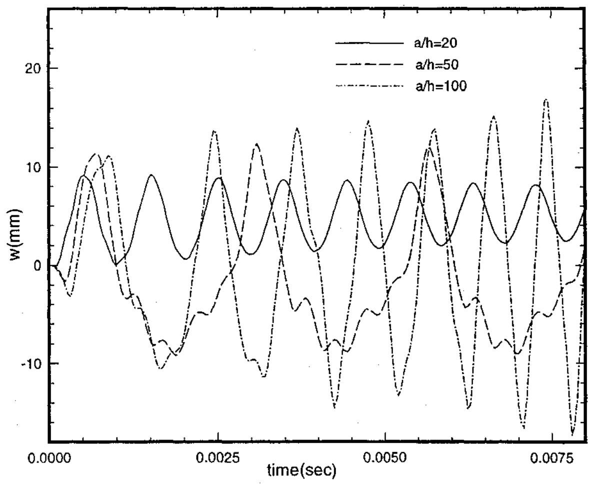

Fig. 3 Effect of the side-to-thickness ratio on the nonlinear transient response of a [0/90 deg] cross-ply square plate.

The thermoelastic version of the laminate constitutive equation can be written as

$$\begin{Bmatrix} N \\ M \\ P \\ Q \end{Bmatrix} = [\bar{D}] \begin{Bmatrix} \varepsilon \\ \kappa \\ \eta \\ \gamma \end{Bmatrix} - \begin{Bmatrix} N^T \\ M^T \\ P^T \\ 0 \end{Bmatrix} \tag{4}$$

where $[\bar{D}]$ is the laminate stiffness coefficient matrix and the thermal resultants are defined by

$$\begin{Bmatrix} N_x^T, & M_x^T, & P_x^T \\ N_y^T, & M_y^T, & P_y^T \\ N_{xy}^T, & M_{xy}^T, & P_{xy}^T \end{Bmatrix} = \int_{-h/2}^{h/2} \begin{bmatrix} Q_{11} & Q_{12} & Q_{16} \\ Q_{12} & Q_{22} & Q_{26} \\ Q_{16} & Q_{26} & Q_{66} \end{bmatrix} \begin{Bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{Bmatrix} (1, z, z^3)\, T \, dz \tag{5}$$

where (α_x, α_y, α_{xy}) are the thermal expansion coefficients in plate coordinates; T is the temperature change from a reference state and has been assumed to have the following distribution: $T(x, y, z, t) = T_1(x, y, t) + (z/h)\, T_2(x, y, t)$.

Finite Element Model

A nine-noded isoparametric quadrilateral element with 63 degrees of freedom is implemented in the present formulation. The present shear flexible model requires only C^0 continuity. The generalized displacements are interpolated over the element and are given by

$$\{\bar{u}^e(x, y, t)\} = \sum_{i=1}^{9} [N_i^e(x, y)]\, \{\bar{u}_i^e(t)\} \tag{6}$$

where

$$\{\bar{u}_i^e\} = [u_{oi} v_{oi} w_{oi} \psi_{xi} \psi_{yi} \phi_{xi} \phi_{yi}]^t \tag{7}$$

and u_{oi}, v_{oi}, etc. are the nodal values.

The element equations are given by

$$[M^e]\{\ddot{\Delta}\} + [K^e]\{\Delta\} = \{F^e\} + \{F_T^e\} \tag{8}$$

where $[M^e]$ is the element mass matrix, $[K^e]$ is the nonlinear element stiffness matrix, $\{F^e\}$ is the element force vector, and $\{F_T^e\}$ is the element thermal load vector. The assembled equations are solved using the Newmark integration scheme in conjunction with the direct iteration technique.

Results and Discussion

Numerical results are presented for the nonlinear dynamic response of metal matrix based laminated plates using a 4 × 4 full mesh of nine-noded isoparametric elements. The following type of simply supported (SS) boundary conditions are considered: $u_0 = w_0 = \psi_y = \phi_y = 0$ at $x = 0$, a and $v_0 = w_0 = \psi_x = \phi_x = 0$ at $y = 0$, b.

The thermoelastic material properties of P55/6051 graphite/aluminum (Gr/Al) considered are: E_1 = 190.0 GPa, E_2 = 48.3 GPa, $G_{12} = G_{13}$ = 17.3 GPa, G_{23} = 16.5 GPa, ν_{12} = 0.28, ρ = 2400.0 kg/m^3, $\alpha_1 = 3.34 \times 10^{-6}$ m/m/°C, $\alpha_2 = 26.1 \times 10^{-6}$ m/m/°C.

Unless mentioned otherwise, a square plate of length 254 mm with a side-to-thickness ratio of 50 is considered for the analysis. The values of T_1 and T_2 are taken as 250 and 500°C, respectively. A time step of 4×10^{-5} s is used for all of the problems considered. Figure 1 shows the linear and nonlinear transient responses of a two-layer cross-ply SS plate. The effect of nonlinearity is to decrease the amplitude and period of the center deflection. Figure 2 shows the effect of the ply orientation on the nonlinear transient response of a SS plate. Of the three lamination schemes considered, the behavior of [0/90 deg] and [45/−45 deg] laminates are found to be similar. Figure 3 shows the effect of side-to-thickness ratio on the nonlinear response of a two-layer cross-ply SS plate. The effect of the thickness on the amplitude of the center deflection is apparent from the figures.

Conclusions

A nonlinear higher order shear deformation theory is used for the dynamic analysis of laminated composite plates. A C^0 continuous finite element model is developed. In contrast to the first-order shear deformation theory, the present higher order theory does not require shear correction factor due to the more realistic representation of the cross-sectional deformation. Numerical results are presented for metal matrix based composite plates which should serve as benchmarks for future studies.

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Hierarchy in the Design and Development of Structural Components: A Preliminary Study

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Introduction

STRUCTURAL components designed and built on the principle of hierarchy have the potential to develop extraordinary capabilities with regard to strength and toughness. Biological structures such as wood, nacre, bone, and cuticle illustrate the potentials of hierarchy extending over several scales in their "design." In this paper an attempt to model simple ropelike structures in tension is discussed from the point of view of examining their suitability for use in composites. Numerical results from a parametric study of a single strand in tension indicate that although an increase in toughness arises due to increased ductility of the element by virtue of its architecture, firm conclusions cannot be drawn until the response in compression is evaluated.

Hierarchy, in the context of this paper, is defined as an arrangement of material and/or structure in which a basic element repeats itself over several increasing scales leading to the desired structural configuration. There is experimental evidence to show that composite panels simulating the hierarchical architecture of wood exhibit extraordinary structural integrity that the material/structure would lack otherwise. The resistance of these panels to impact loads was demonstrated to be far superior to equivalent aluminum panels.[1] A result of the material/structural architecture of hierarchy is that any induced damage is resisted by a series of interruptions in the progress of such damage leading to a tortuous crack path. It is this basic observation that influences consideration of hierarchical elements for structural components.

Ropelike structures are, perhaps, the oldest structural components that have withstood the tests of time in mining, marine, and elevator industries. Our interest is to examine such elements for use as fibers in advanced composites for a wide variety of uses where impact loads may be particularly significant. This paper is an attempt at a preliminary examination of the structural characteristics of such strands. This study does not include the important consideration of the fiber matrix interface that the inherent undulations in the filter architecture are ultimately likely to offer a better interfacial bonding. The use of multimaterial helically wound fibers is expected to add significantly to the design space for new materials. Although use of ropelike fibers with different materials for the core and outer layers is not a new concept, the authors be-

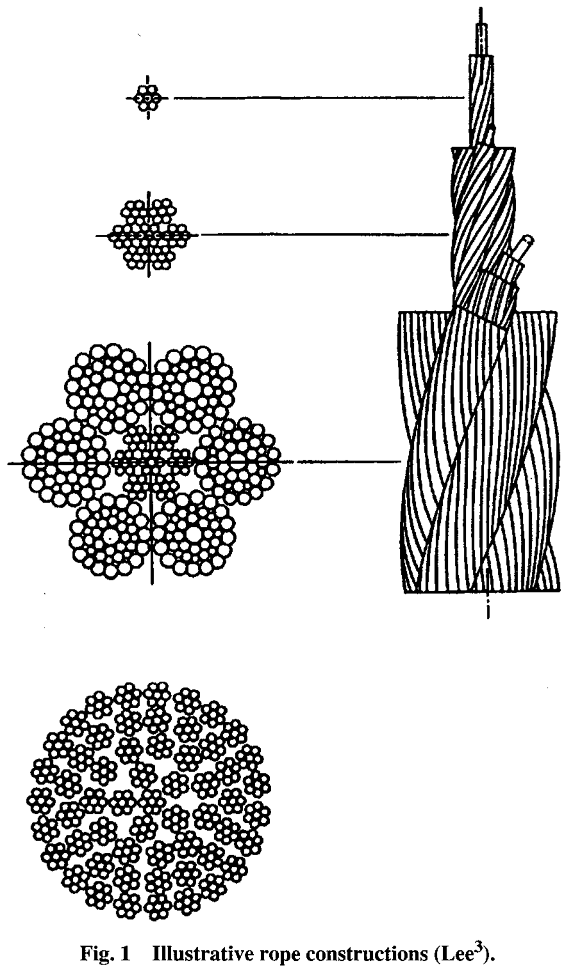

Fig. 1 Illustrative rope constructions (Lee[3]).

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